AMYGDALA $z \mapsto z^2 + c$

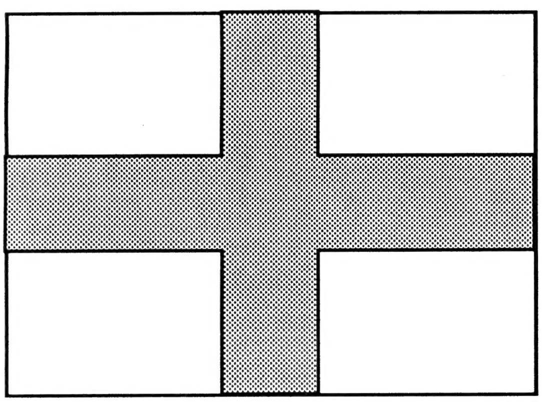
A Newsletter of Fractals & $\mathcal{M}$ (the Mandelbrot Set) Issue #22

December 2, 1990

IN THIS ISSUE...

In *A Mandelbrot Set Lecture Tour*, the brothers Philip — Dave and Ken — report on the Mandelbrot set lectures that they gave at a series of universities in October 1989.

In *Alternative Mappings of Polynomial Julia Sets*, Ian Entwistle shows how to construct some remarkable Julia sets based on polynomials. Ian's approach has three unusual aspects. First, the maximum iteration count is taken very small — never more than 12. Second, the loop is terminated when an iterate escapes from a *square* of side $\sqrt{10}$ centered on the origin, rather than from the familiar circle. Third, and most novel, the "coloring" of a point is based not on its iteration count, but on how it escapes from the square. Referring to the following figure, if the escaping iterate lies within the shaded cross it is colored black; otherwise white. The cross is centered on the origin, and



extends to infinity in all four directions. Its width is $\sqrt{10}$ for Figures 1-2 and 10 for Figures 3-4.

THE SLIDES (S22)

#1858 was produced by Stephen Szilagyi. #3211 was produced by Ken Philip, and is described below in the article, A MANDELBROT SET LECTURE TOUR. #2317 and 2318 were produced by Rollo Silver, using Dave Platt's *MandelZot* program. Those images were saved on diskette as Macintosh PICT files and made into slides using an Agfa Slide Writer film recorder. They all have dwell limit 4095, escape radius 2.5, and pixelation 1024 x 683 — which I call *warp 19.4*.

1858 (Stephen Szilagyi): This is slide #18 of 20 in Stephen Szilagyi's new zoom sequence (SZ4). Sorry, no details are available.

3211(Ken Philip): A tightly-wrapped multi-turn octuple spiral in the sea horse tail above the east Sea Horse Valley radical with a cycle number of 51. This type of many-turned spiral

(Continued on page 2)

CONTENTS

may be found by looking at the center of 'double spirals' taken from the inner loops at the outer ends of the tails of sea horses.

Produced with MandelZot. Center = –0.7485·8089·5602·46 + 0.0630·6469·1779·448*i*; magnification = 2.37×10^{12}; maximum dwell = –7,000.

2317 JDJ: This is the John Dewey Jones midget, once again. It may be compared with JDJ's slide #37 (distributed with Amygdala #2), and with #2314 (distributed in slide set #20). Like #2314, #2317 has less magnification than #37; it differs from #2314 and #37 in having ER (escape radius) = 2.5 instead of ER = 2. The most striking visual difference between ER = 2.5 and 2.0 is that the lobed structures surrounding the central midget become spiky, and the extended undulations extending horizontally and vertically become damped. In the subviews described below, this damped undulatory pattern gives rise to structures suggestive of talons, with the undulatory nodes suggesting joints. Center = –1.99638 x 3.6×10^4.

2318: **JDJ1.** This is a detail of #2317, up the imaginary axis, with RM (relative magnification) = 5,530 relative to #2317. Displayed with a MandelZot custom CLUT, it has a pale cyan ground on which appears a bronze cross form containing a yellow-green plate with talonic extensions having in its middle a green triple hammer. Center = -1.9963·779183 + 0.0000·198593*i* x 1.99×10^8.

A MANDELBROT SET LECTURE TOUR

A. G. Davis Philip — *Union College, Schenectady*

Kenelm W. Philip — *Univ. of Alaska, Fairbanks*

In October, 1989, we gave a series of lectures, "An Introduction to the Mandelbrot Set", at Cornell University (Astronomy Dept.), Wesleyan University (Campus-wide Evening Talk), Yale University (Astronomy Dept.) and Union College (one of the Minerva Lecture Series). We have been working on aspects of the Mandelbrot Set for over two years and have collected a large file of images created on IBM PCs and a Macintosh II. The main program used on the IBM side was Freeman's (Vancouver, Canada) V63Mbrot, which is a program written specifically for use with the ATI VGA Wondercard. It produces pictures at a resolution of 800x600 pixels in 256 colors. The Windows version of Fractal Magic (by Sintar, Bellevue, Washington) has been used (at 1078x768 pixels in 256 colors), but until Version 3.0 of Windows comes out early in 1990 the pictures are not easily edited in color. Some of the pictures were made with the EGA version of Fractal Magic (350x640 with 16 colors).

On the Macintosh side the main program has been Platt's (Palo Alto, CA) MandelZot, which produces 256-color pictures at 640x480 pixels on a 13-inch monitor. Woodhead's (Ithaca, NY) MandelColor 5.5 and Lankton's (Boulder, CO) Ani-Mandel have also been used, both yielding 256 colors (or nearly) at 640x480 pixels. In all cases, the color slides were made by direct photography of the monitor (using 200 mm lenses on Nikons). For ASA 64 Kodak film the ASA guide number was set at 100.

The talk was presented in three parts. Part I was a "Tour of the Bug" in which we showed color slides from around the edge of the $\mathcal{M}$-Set at low magnification and dwell. The major features were identified and described. Part II was devoted to an explanation of how the $\mathcal{M}$-Set can be generated on a personal computer, and discussion of the parameters used in the calculations. (See "Wormtracks" by KWP, AMY #14; and "The Taming of the Shrew", by AGDP and KWP, AMY #19 for additional details.).

Part III was a more detailed exploration of the major features of the M-Set (such as the Spike, Sea Horse Valley, Elephant Valley, Radicals and tendrils) at much higher magnifications and dwells. Certain objects were singled out for emphasis, as follows:

1. A series of 16 slides of one region in Sea Horse Valley (a midget surrounded by four circular features and enclosed in an approximately square boundary) produced by 6 different programs and with different color assignments and dwells. Major changes in appearance illustrate the importance of aliasing effects in the presentation of complex $\mathcal{M}$-Set images.

2. A series of slides of $\mathcal{M}$-Set features which mimic astronomical objects, such as spiral galaxies (both normal and barred), globular clusters, sunspots and solar prominences.

3. "Elephant Valley", as we prefer to call the main valley at the east end of the $\mathcal{M}$-Set (because of the endless stream of elephant-like features that appear to be walking out of the valley), and features associated with it.

4. "Pinwheels", which are radially symmetric features around midgets at the centers of "galaxy clusters" (small condensations on tendrils radiating out of midgets).

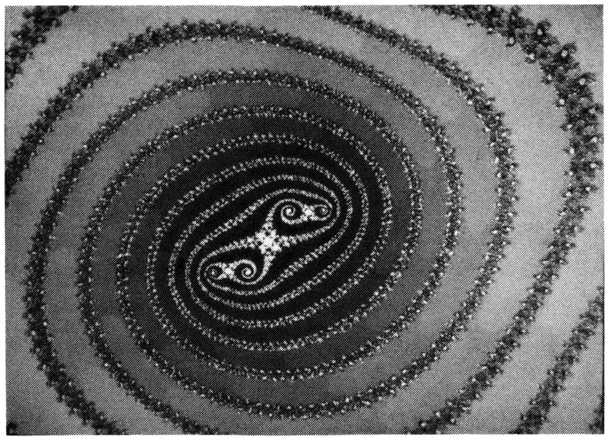
5. Double spirals, which are features that link sections of Sea Horse tails or Elephant trunks (and are found in similar situations throughout the $\mathcal{M}$-Set. (See The Beauty of Fractals, Maps 45 and 47, for a typical double spiral). Recently we have found double/double spirals in which the small end spirals of the double spiral are themselves double spirals. Some of these features may be seen in this month's slide selection, two of which are taken from the lecture tour slide set.

At the end of the talk we briefly discussed the sets generated by equations other than $z \leftarrow z^2 + c$ and showed a number of examples. Several Julia Sets were shown and their use in analyzing $\mathcal{M}$-Set structures was explained.

At each institution we met people who were investigating the $\mathcal{M}$-Set with programs they had written or obtained, and we have had many interesting discussions. Interest in fractal images seems to be a widespread phenomenon, which is by no means limited to math and computer science departments — as was shown by the interest expressed by a number of artists who were present at the talks.

Slide set

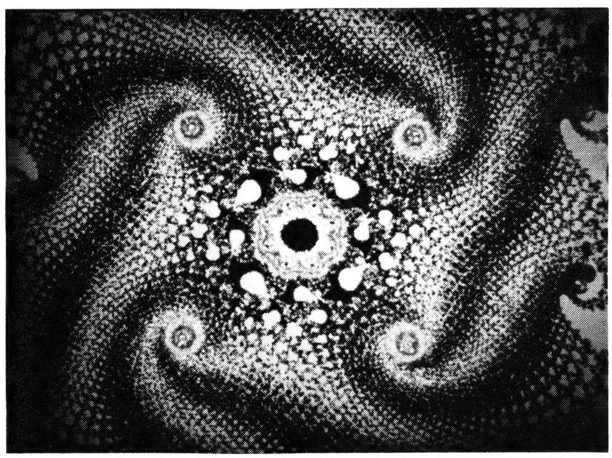
1. The first double/double spiral seen by AGDP in Sea Horse Valley. It is located[1] at $\Re$ = –0.7485·8089·5648·7641, $\Im$ = 0.0630·6469·1777·6402 and was plotted with a maximum dwell of 20,000.



The side length in real coordinates is 10^{-15}. This picture was computed by Freeman's V63mbrot.

This slide, Amygdala #818, is part of the slide supplement distributed with this issue of the newsletter.

2. One of the 16 versions of "SHVRB58" discussed in section 1.

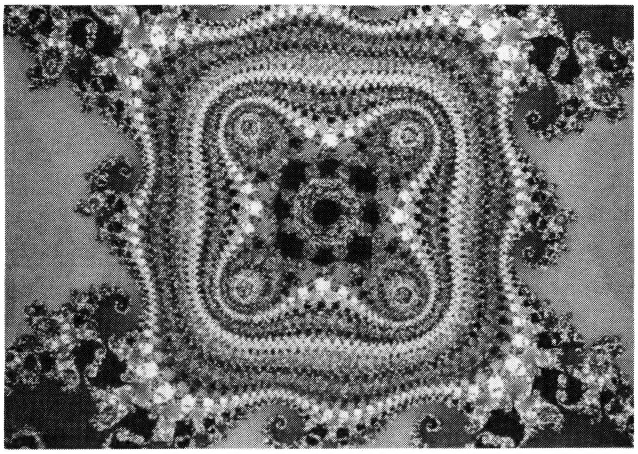


This picture was produced by the EGA version of Fractal Magic. It is located at $\Re$ = –0.7459·0609·15, $\Im$ = 0.0984·8530·35 and has a maximum dwell of 3,825 and a magnification of $1.67{\times}10^9$. This version of the picture accentuates the background spiral structure. This slide, Amygdala #803, is part of slide supplement #20.

3. Another view of "SHVRB58", produced by MandelZot. $\Re$ = –0.7459·0609·158, $\Im$ = 0.0984·8530·361, maximum dwell is 3,825 and the magnification is $1{\times}10^9$. This image accentuates

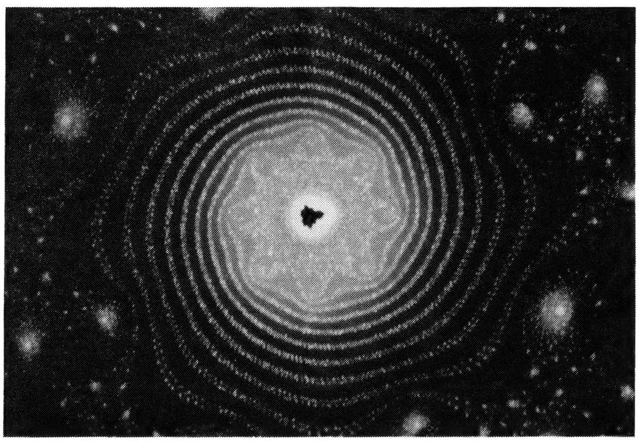
1. To aid in reading these long strings of digits, I have introduced a relatively inobtrusive center-dot to break the string into groups of four digits. I hope this helps rather than hindering. — *Ed.*

the four-lobed pattern in the center, and shows a series of rectangular color bands in the outer portions of the image.



This slide, Amygdala #3210, is a color variant of #535, part of slide supplement #18.

4. A tightly-wrapped multi-turn octuple spiral in the sea horse tail above the east Sea Horse Valley radical with a cycle number of 51.



Produced with Dave Platt's MandelZot: $\Re$ = –0.7485·8089·5602·46, $\Im$ = 0.0630·6469·1779·448, maximum dwell = 7,000, and magnification = 2.37×10^{12}. This type of many-turned spiral may be found by looking at the center of 'double spirals' taken from the inner loops at the outer ends of the tails of sea horses.

This slide, Amygdala #3211, is part of the slide supplement distributed with this issue of the newsletter.

Slides #1-2 are by AGDP using IBM computers, #3-4 are by KWP using a Mac II.

ALTERNATIVE MAPPINGS OF POLYNOMIAL JULIA SETS

— Ian D. Entwistle

When the quadratic transformation $f(z) = z^2 + \mu$ is iterated in the complex μ plane the resulting mapping is the well known Julia set. The sequence $f(z_{n-1})$ may diverge or converge. Divergent points in the μ plane are usually identified by the iteration value for which $|z| > 2$. This definition of the escape radius determines that the curve for an iteration value $k = 1$ is a circle. The iterative process is thus terminated when either k attains a large predetermined value or $|z^2| > 4$. High rates of divergence towards infinity are found for points with low values of k, and so interesting chaotic patterns are mostly found close to the points in the Julia set. Thus, generating the most pleasing maps requires substantial CPU time. In calculations where both real and imaginary parts of z are iterated separately, the real part is the major component which diverges towards infinity. Alternative mappings can thus be obtained by testing the real and imaginary parts of z using various strategies and test values. Some of these methods have been previously reported (See References); their main advantage is that interesting fractal patterns can be produced with fewer iterations.

Most studies of polynomial functions utilise the iteration value to control the mapping colour. There is however a wealth of information hidden within the actual data produced for each point in the complex plane by iteration. Figures 1 to 7 are obtained using such data. These maps differ from classical Julia sets, since all the points of interest are obtained at low values of k. Typically values of $k > 12$ result in the disappearance of the fractal pattern giving uniform blank areas on the maps.

The method is illustrated by application to the transformations $f(z) = z^n + \mu$ for $n = 3, 5, 7$. In order to allow Amygdala readers to generate fractals of the type illustrated, the description assumes that you have a listing in BASIC for generating Julia sets.

Firstly, the algebraic equations used are derived using the standard method of relating complex numbers (these have been described in earlier issues of Amygdala). For interest and to save

everyone the laborious algebra, the real and imaginary parts of $z^n = (x+iy)^n$ are listed in Table 1:

n	Real	Imaginary
3	$x^3 - 3xy^2$	$3x^2y - y^3$
5	$x^5 - 10x^3y^2 + 5xy^4$	$5x^4y - 10x^2y^3 + y^5$
7	$x^7 - 23x^5y^2 + 35x^3y^4 - 7xy^6$	$7x^6 - 35x^4y^3 + 23x^2y^5 - y^7$

Table 1

Students of mathematics may observe that the coefficient of the mixed seventh power term is incorrect for the expansion of $(x+iy)^7$. The correct value (21 instead of 23) gives less interesting mappings. On most microcomputers the `FOR-NEXT` loop is faster than other control loops for simple tasks, so the BASIC listings use them. Note that in each case `IT%` is the number of iterations chosen as maximum, and that instead of testing z^2 directly, the squared values of the real and imaginary parts of z are both tested in order to determine when the iteration is complete. The second test then selects for actual values in order to determine the colour value for mapping. The illustrations were printed on an Epson FX1000 at 216 dots/inch down and 120 dots/inch across. You should note that if you calculate at 240/inch across you will lose alternate dots if you then print at 240/inch. This occurs because of the way the Epson overlaps the dots in quadruple printing mode. Disbelievers should try it after they have read the printer manual!! Some use is made of the iteration value to control the printing of the outer edges of the fractal images.

In the listings, points for which `TI%=2` are not printed.

As can be noted from the listings, the colour of the points is controlled by the second test, which gives either 0 or 1 to print white or black. Other colours can be introduced by adding further tests for other values of `X2` and `Y2`. Controlling colour on the screen by iteration value produces some interesting challenges. It is easier to edit the colours if you have global or local colour control on your desktop or painting utility. The real problem is simply that it is hard to determine which parts of the fractals are derived from any particular iteration. Some useful variations can be obtained by changing the values of the escape radius in both tests. Other examples of Julia sets using this approach can be found in Ref. [1].

Listing 1. z^3**:**

```
100 FOR TI% = 1 TO IT%
200   X2 = X*X : Y2 = Y*Y
300   IF X2 > 10 OR Y2 > 10 THEN
        GOTO 700
400   Y = Y*(3*X2-Y2) + IC
500   X = X*(X2-3*Y2) + RC
600 NEXT
700 IF X2 < 10 AND Y2 < 10 AND
        TI% # 2 THEN PRINT
```

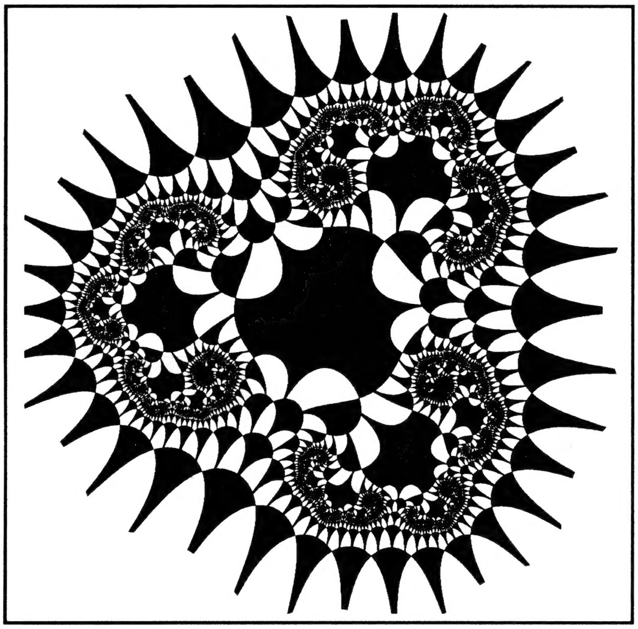


Figure 1— $z^3 + c$

Listing 2. z^5**:**

```
100 FOR TI% = 1 TO IT%
200   X2 = X*X : Y2 = Y*Y
300   IF X2 > 10 OR Y2 > 10 THEN
        GOTO 700
400   X3 = X*X2 : Y3 = Y*Y2 : XY = X*Y
500   Y = Y3*(Y2-10*X2) + 5*X3*XY + IC
600   X = X3*(X2-10*Y2) + 5*XY*Y3 + RC
700 NEXT
800 IF X2 < 10 AND Y2 < 10 AND
        TI% # 2 THEN PRINT
```

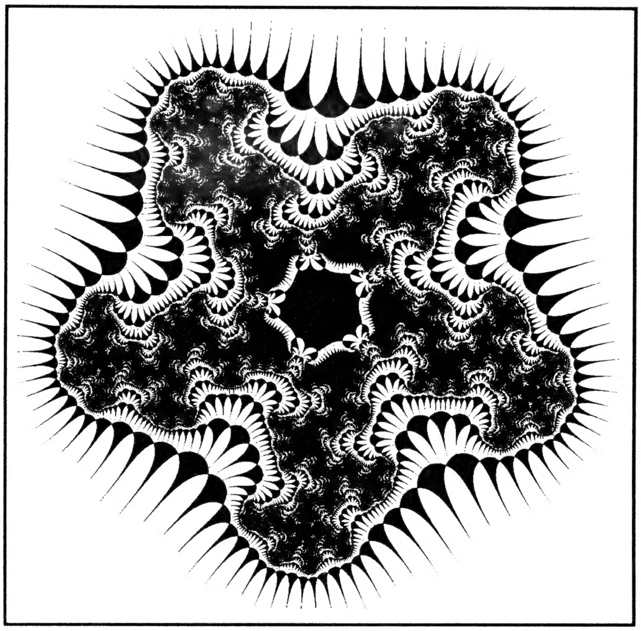


Figure 2 — $z^5 + c$

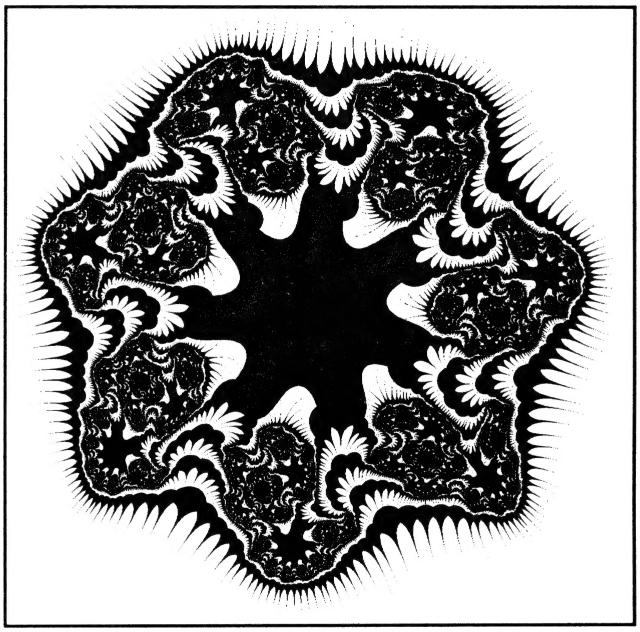


Figure 3 — $z^7 + c$ **variant**

Listing 3. z^7:

```
100 FOR TI% = 1 TO IT%
200   X2 = X*X : Y2 = Y*Y
300   IF X2 > 10 OR Y2 > 10 THEN
         GOTO 700
400   X4 = X2*X2 : Y4 = Y2*Y2
500   X7 = X*Y2*(35*X2*Y2-23*X4-7*Y4) +
         X4*X2*X + RC
600   Y7 = X2*Y*(7*X4-35*X2*Y2+23*Y4)
         - Y4*Y2*Y + IC
700   X = X7 : Y = Y7
800 NEXT
900 IF X2 < 100 AND Y2 < 100 AND
         TI% # 2 THEN PRINT
```

For these listings **TI%** = iteration value, **IT%** = maximum iteration value, **RC** = real part of μ, **IC** = imaginary part of μ. The various variables used for powers of **X** and **Y** are included in order to minimise multiplications. Inputs for **X**, **Y**, **RC**, and **IC** should be of the same form as for a typical z^2 Julia set.

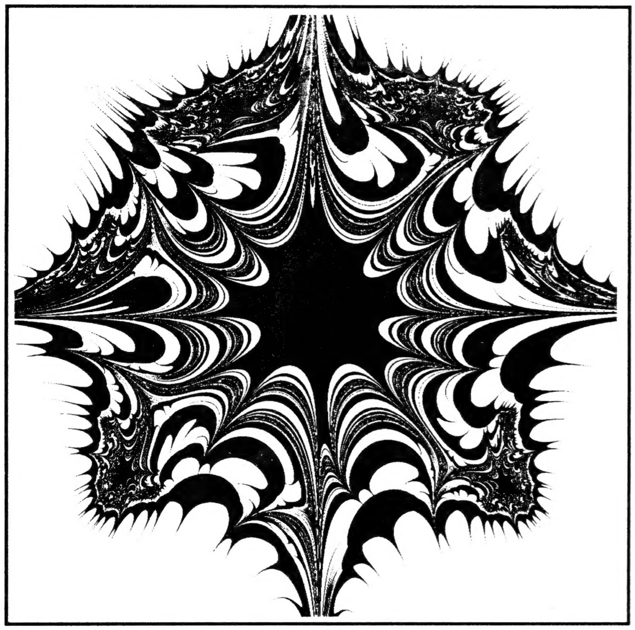


Figure 4 — $z^8 + c$ **variant**

Table 2 (next page) lists the input data for figures 1 to 4. The other figures 5-7 are included to illustrate the variations that occur with values quite close to those quoted. Try the range –1 to 1 for each of **RC** and **IC**:

Figures 1-4, whose window limits are given in Table 2, were run at 1000 x 1890 pixels. The other examples were obtained at 360 x 630 pixels. Several examples not shown have been run at 1620 x 2800 and then photographically doubled in size. The resulting prints look sharper than a laser print. Figure 4 results from adding an extra term arbitrarily to the equations for figure 3. Thus x^7y

replaces the first real term and xy^7 the last imaginary term. Note that figure 4 now has eight lobes.

Fig.	min/max X	min/max Y	RC/IC	#iter
1	-1.5/1.5	-1.5/1.5	0.6/0.05	10
2	-1.3/1.3	-1.3/1.3	0.32/-0.8	12
3	-1.2/1.2	-1.2/1.2	-0.3/0.7	12
4	same as 3		-0.25/0.8	12

Table 2 — Window Limits

References

[1] .D. Entwistle, Julia set art in the complex plane. Comput. & Graphics (13,3) 1989. pp. 389-392.

[2] C. Pickover and E. Khorasani, Computer Graphics Generated from the Iteration of Algebraic Transformations in the Complex Plane. Comput. & Graphics (9,2) 1985. pp. 147-151.

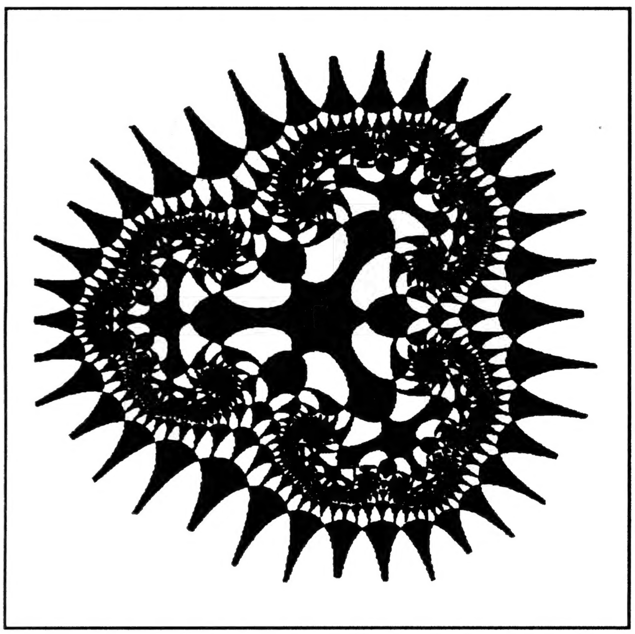


Figure 5

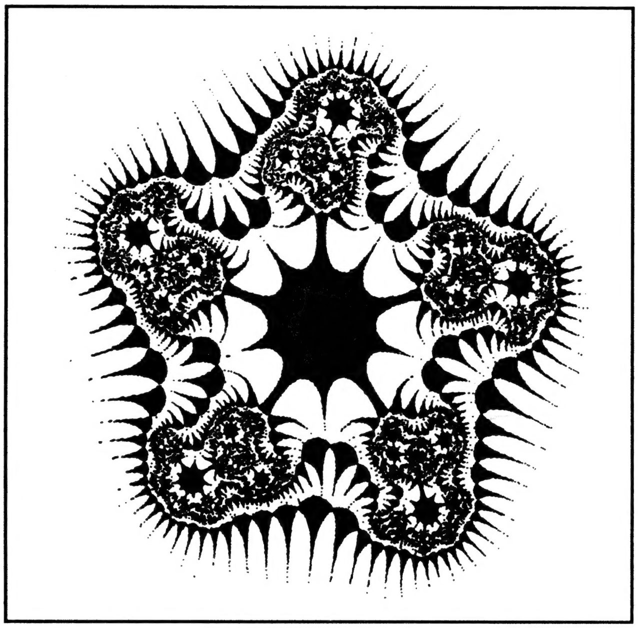


Figure 6

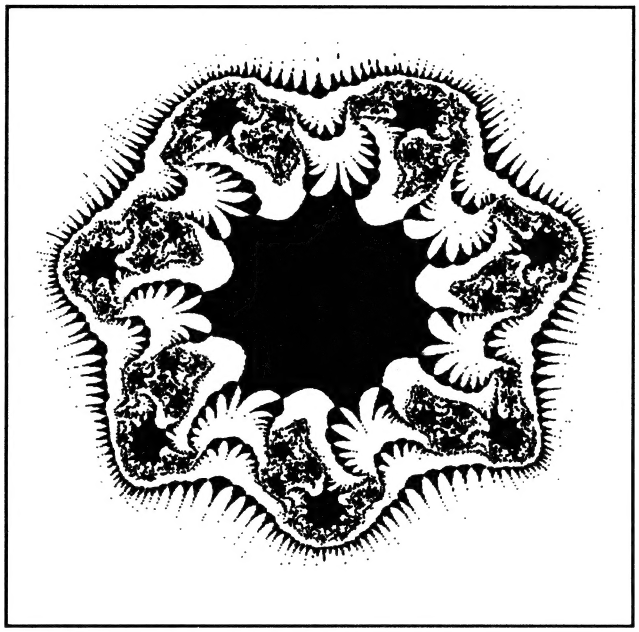


Figure 7

EMAIL, ANYONE?

Persons wishing to communicate with me, Rollo Silver, Amygdala editor & publisher, via E-mail, can do so through my Internet address:

rsilver@LANL.gov

GLIMPSES OF A FUGITIVE UNIVERSE

— *Rollo Silver*

I have created a set of twelve beautiful[1] fractal images, called *Glimpses of a Fugitive Universe,* which I am offering (in whole or in part) as high-quality Cibachrome prints at Warp-21.4 resolution in sizes 8x10", 11x17", and 16x20".

To give you an idea of what they look like, I am offering these images as a set of twelve 35mm color slides, at Warp-19.4 resolution. The price is $15.00 for the set of twelve slides, plus P&H. See the latest *Price List / Order Form* for details.

1. An objective evaluation! See G. D. Birkhoff's *Aesthetic Measure*.

FRACTAL CONFERENCE 1992

— *Rollo Silver*

I assert that it would be nice to have a technical conference about fractals in the Taos area in the summer of 1992. This is a trial balloon, to see who is interested in coming, submitting a paper, or helping to organize it.

Since my time is drastically overcommitted as is, I couldn't possibly organize it, except to handle local arrangements. Therefore the *persona sine qua non* is the conference organizer. Anyone interested? JDJ?

1991 FRACTAL CALENDARS

— *Rollo Silver*

FRACTAL COSMOS 1991

Amygdala has been offering Amber Lotus' Fractal Calendars for the past three years. The 1989 and 1990 calendars featured fractal images from H.-O. Peitgen and his group.

The new 1991 calendar has images created by a variety of artists, most of which are contributors to Amygdala: Ken Philip (frontispiece, January, May, June, October), John Dewey Jones (February, November), Rollo Silver (April), Ian Entwistle (August, September, December).

My April image is also the calendar cover, and is one of my high-resolution images — #2301 (slide set #20). That same image is also available as a set of twelve greeting cards with mailing envelopes.

See the *Amygdala Price List & Order Form* for details.

LOYLESS 1991

James E. Loyless has been producing fractal black and white calendars for several years. Now his 1991 calendar is available. Its full open size is 8.5 x 14 inches. You can get this calendar by sending $7.00 ($7.50 outside US) to:

James E. Loyless
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NEW SZILAGYI ZOOM SEQUENCE

Stephen Szilagyi has come up with a new zoom sequence of twenty slides, which is available as *SZ4* (See the *Amygdala Price List & Order Form* for details). One image from this sequence appears as slide #1858 in the current slide set (#22).

CIRCULATION

As of December 2, 1990 Amygdala has 730 subscribers, 266 of whom have the supplemental color slide subscription.

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For 51 of you subscribers out there this is the last issue of your current subscription. I urge you to use the enclosed form to renew your subscription promptly to avoid missing anything.

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MERZ

Fractal software, publications, and other stuff available from individuals and small companies.

Dada was a movement of artists and writers in the early 20th century who exploited accidental and incongruous effects in their work and who programmatically challenged established canons of art, thought and morality.

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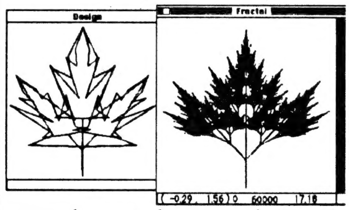

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